Geek **Hub**

What do engineers see in coffee grounds? FloEFD Validation applied to the tasks of geometrical optics

By Dr. Svetlana Shtilkind, Dr. Andrey Ivanov and Maxim Popov, Mentor a Siemens Business

Our life is full of light. Inside or outside, near or far, light surrounds us everywhere and penetrates even in the dark places. Light expresses itself in many ways. One of them is shiny patterns produced by reflection and refraction of rays from complex surfaces (Figure 1).

They are called caustics – the curves or surfaces which are tangent to patches of rays. Caustics outline the boundaries of maximum radiative flux.

The object of our investigation is well-known and one of the most beautiful caustic. Inspired with amazing properties of light, we've focused its rays in the shape of the heart or cardioid in terms of math.

The origins of this work are following to the kitchen and arising on the bottom of a cup (Figure 2).

Side irradiation and cylindrical specular surfaces of cup are necessary conditions to obtain the cardioid. Therefore you can easily observe cardioid inside glasses, cups of coffee, pans and other equal tableware. Let us consider how it appears.

Cardioid is an algebraic curve of 4th order. Its polar equation is $r = 2a \cdot (1 - \cos \phi)$. The coefficient a is based on the geometrical interpretation of curve. On the one hand cardioid can be defined as trace of point M which belongs to the circle of radius a. This circle is rolling without slipping on another circle of radius a, starting from the point O (Figure 3a). On the other hand cardioid is an envelope of circles built on the generatrix circle of radius a with radiuses OP (Figure 3b).

Mathematical substantiation of our simulation is based on the geometrical law: cardioid is an envelope of light rays emitted from the point on the circle and reflected specular from circle's boundary (Figure 4).

In other words cardioid is a caustic of circle with a light source, placed on its boundary. This law defines the distribution of radiant flux after it has been reflected from cylindrical surface (Figure 5).

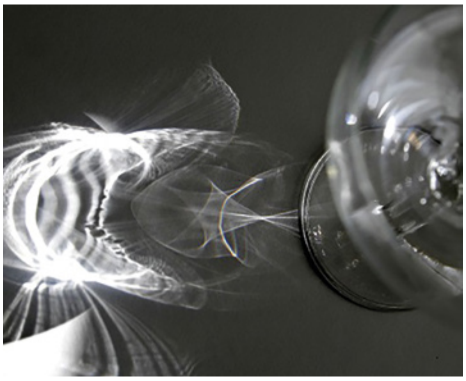

Figure 1. Caustics produced by a glass [1]

Figure 2. Cardioid inside a cup [2]

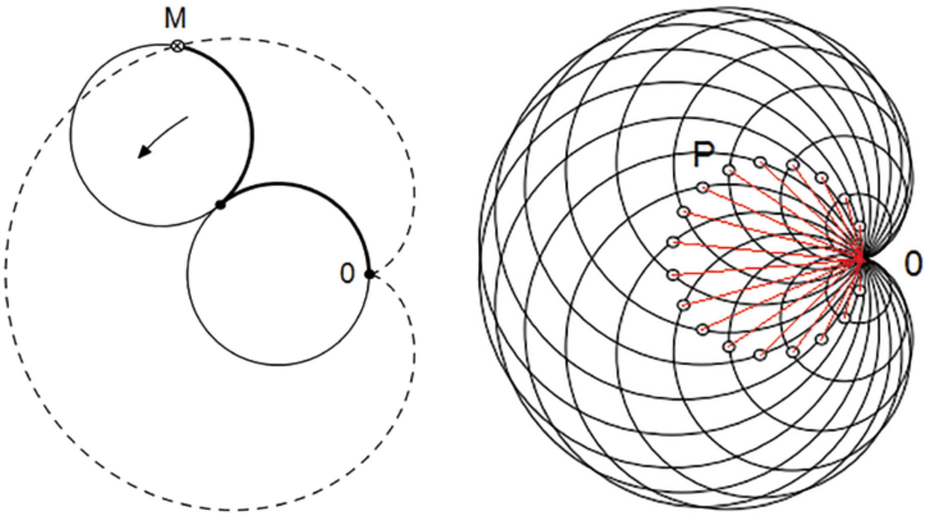

Figure 3. The ways of how the cardioid can be created [2]

The model

For our simulation the real cup with complex conical reflecting surface was chosen (Figure 6).

The model of a cup has been constructed partially filled with some liquid (coffee is preferable), but reflecting and refracting properties of this wasn't taken into account. Its volume was defined like solid, impermeable for light. Walls of cup were also interpreted as opaque. The surface of liquid was treated like a screen which accumulates incident rays reflected from cup's interior. Rays intensity can be evaluated through distribution of net radiant flux.

The source of light was defined as diffusive. Its size was close to point to exclude the influence on the propagation of light.

Influence of source location

Obviously the location of the source determines the distribution of radiative flux on the screen. To find the location of source on which light will be focused into cardioid, a set of preliminary calculations were performed. The source was examined in 16 positions, differed by horizontal and vertical coordinates (Figure 7).

The growth of both coordinates was started from the point 0 on the cup edge. Spatial step along X was 15 mm, along Y – 40 mm, radius of cup edge – 50 mm. Fig.8 illustrates the results of calculation: distribution of net radiant flux on the screen.

It is easy to see how the location of source affects the distribution of radiative heat flux. When the source is in the low position (Y=0) and far enough from cup edge (X>1), the screen is fully shadowed. But if even small portion of light has reached internal surface of cup, the rays are being focused in the shape of cardioid. The size of cardioid grows as the source moves from cup edge.

To demonstrate the influence of height of reflecting surface into optical image we've simplified our model and explored reflection from cylindrical walls. The two configurations have been explored: at the first case walls height was close to zero – the model abstraction to organize reflection on the plane of bottom only. At the second case reflections took a place along the wall's height. The results of calculations are presented in Figure 9.

It is easy to see how the distribution of radiative flux changes in accordance with height of reflective walls. If rays are reflected from «boundary of the circle» (case a), we've got sharp cardioid. If the reflections occur in different height (case b), the image becomes more complex and includes secondary maximums.

Now we can choose the proper location of source to reproduce the cardioid of desired configuration. The results of calculations are very close to the real observed picture (Figure 10 and 11).

To confirm correspondence between the calculated caustic and cardioid, let us

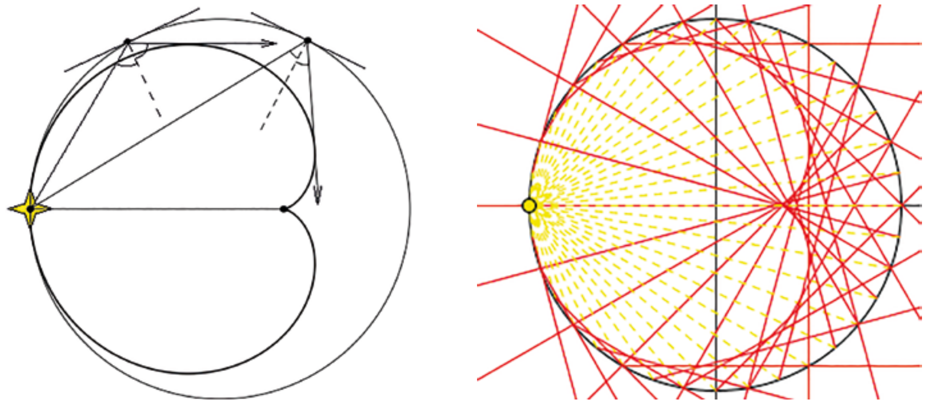

Figure 4. Geometrical interpretation of cardioid [3, 4]

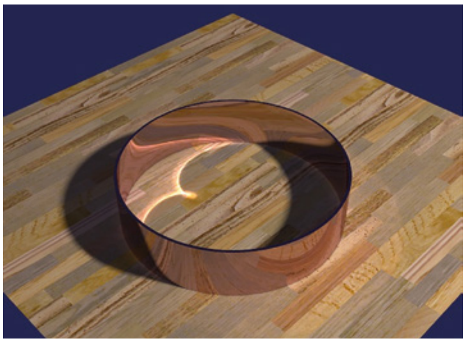

Figure 5. Cardioid produced by cylindrical reflecting surface [3]

Figure 6. The model of a cup

compare the maximum of net radiant flux with the curve defined by equation r = 2a·(1-cos ɸ). The two situations were considered: the source is located at X=2, Y=3, second – at X=0, Y=2. It is easy to see that the first results corresponds to cardioid with coefficient a=0.0588, and the second results – to cardioid with a=0.0480 (Figure 12).

Both results have reproduced accurately the cardioid of two types, which differs from each other in the source location.

3D cardioid

Starting from the two-dimensional image on the bottom of the cup, let us expand our vision of cardioid in 3D case. The next example presents quite an original diagram of radiation visibility. Radiation is being detected on the far field from the spherical heat source, made from two equal parts with different temperatures (Figure 13).

Let us imagine the circulation around heat source in the far field. The objective of the circulation is to collect the values of radiation flux in each point of orbit. Orbit belongs to the one of three planes: XY, XZ and YZ. The results of such investigation predicted with FloEFD in polar coordinates are shown in Figure 14. Radiation flux detected in YZ plane is a circle, because of uniform impact of each point of heat source. Distribution of energy crosswise source in XY and XZ planes

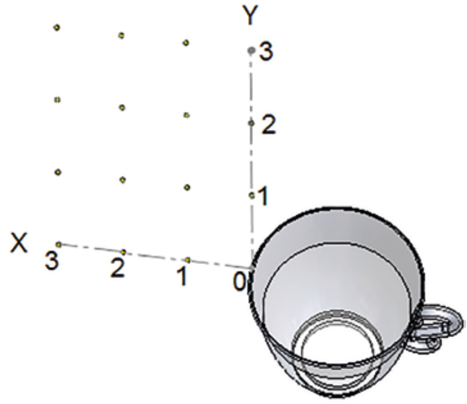

Figure 7. Source locations, gridded in coordinates thX0Y

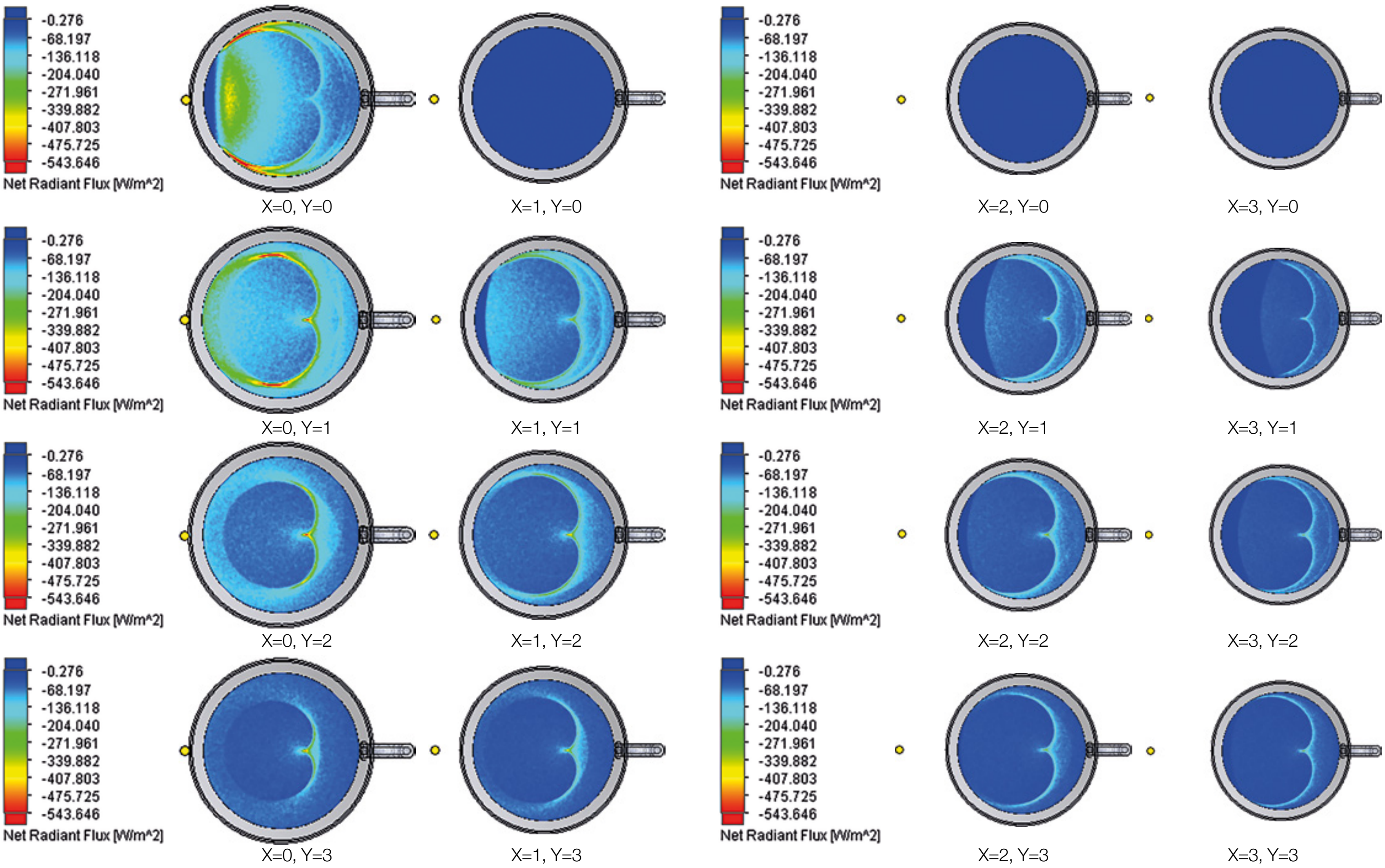

Figure 8. Distribution of net radiant flux on the screen: top view. The location of source is marked with yellow point.

Figure 9a. reflection from the wall: height is close to zero

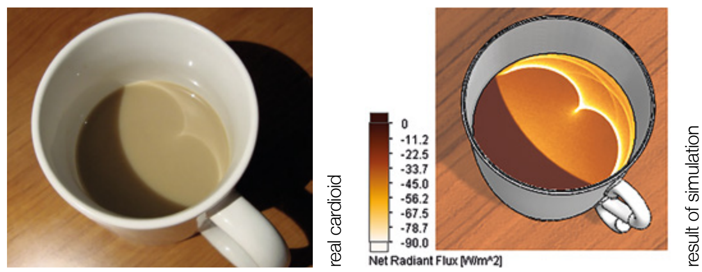

Figure 10. reflection from the wall: height is close to zero

Figure 9b. reflection from the wall: height is nonzero

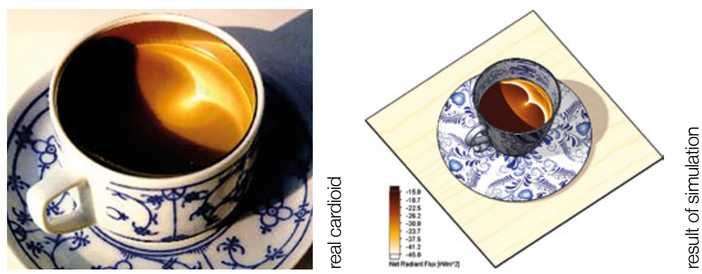

Figure 11. Reality and the results of simulation [5]

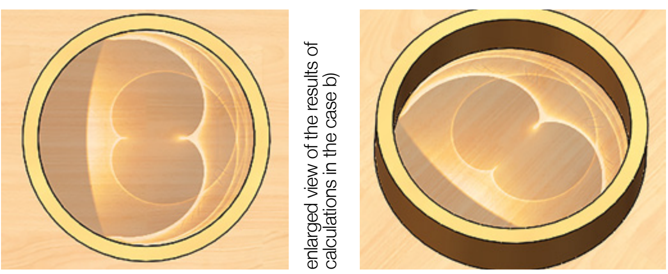

Figure 9. Cardioids produced by the walls of different heights

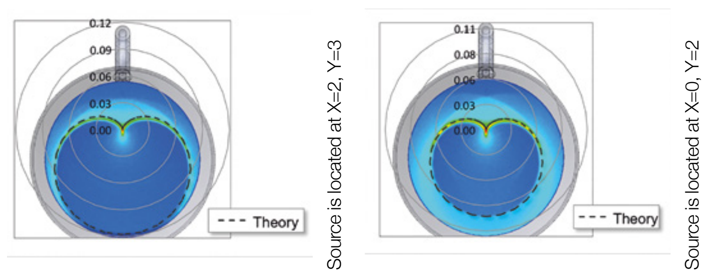

Figure 12. Comparison between the shapes of calculated caustics and analytical curves

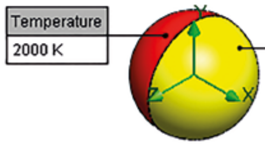

Figure 13. Spherical source with nonuniform distribution of temperature

depends on the angle of view. It changes monotonically from the coldest point (X=+ Rv, Y=0, Z=0) to hottest point (X=- Rv, Y=0, Z=0). Its shape corresponds to cardioid with coefficient a=0.00057.

By the way an ordinary example of such a source is a simple lamp. Its radiative flux, measured in the equally-spaced positions, creates a cardioid (Figure 15).

Thereby radiative flux produced by non-uniform heat source, measured on the surface of imaginary sphere and presented in polar coordinates, takes the shape of 3D-cardioid (Figure 16).

3D-cardioid can be a result of propagation not only light rays, but even sound waves (when they are treated as rays – linear acoustics). The cardioid microphone has a special name because of its sound pick-up pattern [6]. Polar pattern graphs visually represent the microphones sensitivity to sound relative to the direction or angle from which the sound arrives. Figure 17 illustrates how sensitive it is to the sounds coming at different angles. Each point of cardioid corresponds to a composition of an angle at which sound enters the microphone and a volume of sound being input.

In our investigation multifunctional CFD-package FloEFD has demonstrated its wide opportunities in the area of precise optics simulation. The models of radiative heat transfer which are taking into account different ways of light propagation enables analysis of complex interference of light. The results of calculations are in perfect agreement with experimental data, theoretical estimates and our living experience.

Taking a look at the bottom of the cup engineers are thinking of the beauty of nature and predictive power of the modern numerical methods. We do believe that calculations with FloEFD can shine a light on the dark issues in engineering.

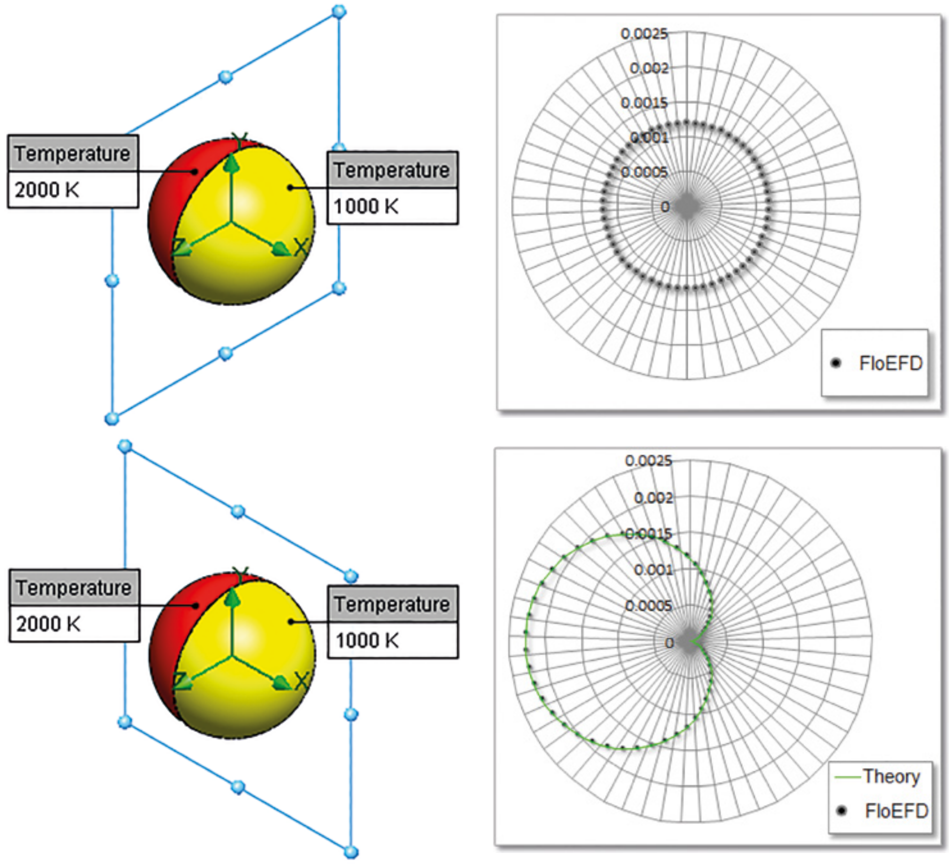

Figure 14. Radiative visibility of the source in different planes

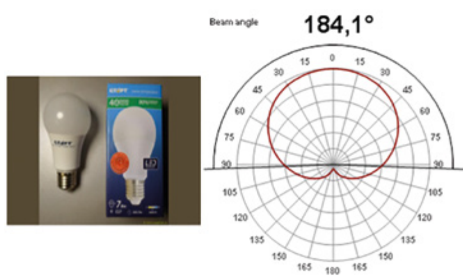

Figure 15. Radiative flux from lamp

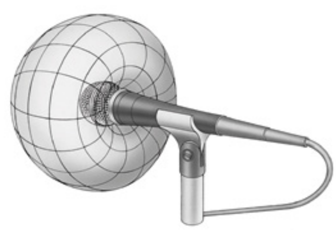

Figure 17. microphone with cardioid pick-up pattern

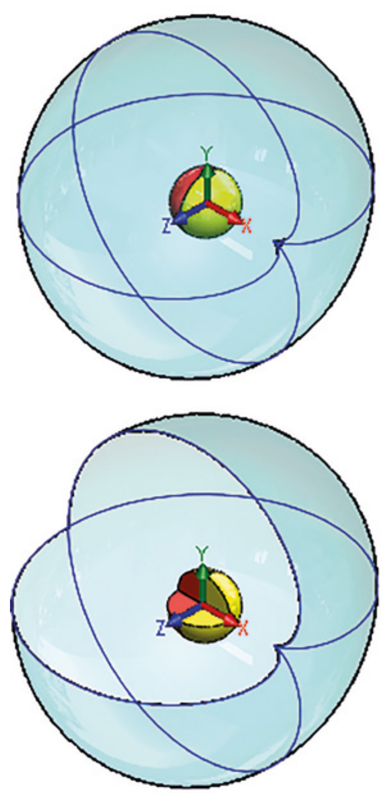

Figure 16. Total radiative visibility of the source

References:

[1] https://ru.wikipedia.org/wiki/%D0%9A%D0%B0%D1%83%D1%81%D1%82%D0%B8%D0%BA%D0%B0

[2] http://prof.pantaloni.free.fr/spip.php?article58

[3] http://graphics.stanford.edu/~henrik/images/metalring.jpg

[4] A.V.Akopian. Geometry of a cardioid. KVANT Journal, №3, 2012 (In Russian).

[5] http://thatsmaths.com/2014/09/25/curves-with-singularities/

[6] http://mathforum.org/mathimages/index.php/Cardioid

ENGINEERING EDGE

Accelerate Innovation with CFD & Thermal Characterization

Mentor®
A Siemens Business

This article originally appeared in Engineering Edge Vol. 7 Iss. 1

Download the latest issue: www.mentor.com/products/mechanical/engineering-edge

You might also be interested in...

Thermal Simulation of a SYN TRAC Powerpack

Edge Devices are Hot for IoT

Distributed Chilled Water Pumping AC System

Reliability Evaluation of Sintered Copper Die Bonding Materials

Virtual Reality and Modeling Tools

Ask the GSS Expert

Simulation of Hydraulic Servo Gimbal Control of a Space Launch Vehicle

How to… Convert CAD Geometry into a FloMASTER Sub-System

A Study of Aerothermal Loads

Development of a Turbocharger with FloEFD

Risk Prevention & Energy Saving in Ships

Interview: Takuya Shinoda, Denso Corporation

Three Thermal Simulation & Test Innovations for Electronics Equipment Design

Series Car Brake Cooling

Liquid Cooling Turned on its Side

Thermal Analysis of an ADAS Camera

Thermal Design Leading the Charge

Understanding Die Attach Thermal Performance

Rapid Beverage Cooling Analysis

Bitcoin Mining: A Thermal Perspective

Designing High Efficiency, Low-Cost Synchronous Reluctance Motors

From Measurements to Standardized Multi-Domain Compact Models of LEDs

A Cool Emulator

Achieving Precision in 1D Dynamic Models of Hydraulic Servo Valves

Using FloEFD as an Engineering Tool

Comparison between Experimental & Computational Results

Design of a High Speed Decoy UAV

Thermal Interface for Pluggable Optics Modules